National Meteorological
Library & Archive
London Road
Bracknell RG12 2SZ

HYDROLOGICAL MEMORANDA No.33

(First Reprint Issued August 1968)

RAIN INTENSITY FREQUENCY RELATIONSHIPS IN BRITAIN

		Page
I.	The Bilham treatment, and its long survival	1
II.	Later developments, and a fresh appraisal of the Bilham treatment's scope	6
III.	Trial of the Gumbel type of treatment	12
IV.	Studies of systematic effects of locality: local or regional proneness to heavy rain	19
V.	Studies of the smoothing effect of averaging over various sizes of area	27

November 1964

D. J. HOLLAND
Met.O.3

ORGS UKMO H

National Meteorological Library
FitzRoy Road, Exeter, Devon. EX1 3PB

METEOROLOGICAL
National Meteorological
Library & Archive
London Road
Bracknell RG12 2SZ
COMMAND

I. THE BILHAM TREATMENT, AND ITS LONG SURVIVAL

It may be recalled that in every volume of British Rainfall since 1925, somewhere near the end of the chapter on "Heavy falls in short periods", there has been a table (numbered 6D for many years) of numbers of days on which specified amounts of rain fell in specified times, an explanation of which was given by E.G.Bilham in a famous article on "Classification of heavy falls of rain in short periods" in the 1935 volume.* The first ten of the annual tables of numbers of days were totalled so as to form a decade's summary table, and although Bilham recognised that a decade would be too short a term to reveal reliably the long-term relationship between intensity and frequency at any individual station he was nevertheless able to cull from the summary table an approximate relationship, for use in England and Wales at least, by simply taking an average over twelve stations - one in Anglesey (Holyhead) and the rest in various parts of England, three of which (namely Camden Square, Croydon, and Kew Observatory) were in what is now Greater London. Somewhat similar averaging was done also over a few stations in Scotland and Ireland, though these were not formally used in culling the aforesaid relationship.

---

* The article was published afresh in 1962 and is accordingly still available from H.M.S.O.

What Bilham was effectively doing was first counting station-days and then dividing by the number of stations so as to get station-days per station-decade ; and in so far as they could legitimately be interpreted as if they were a typical station's numbers of days per decade they were acceptable as a measure of frequencies over not too short a term. The variables to which they were then related were in essence the following two :-

(1) daily peak value of smoothed intensity (the process of smoothing being that of taking a running mean), and
(2) duration or size of time-step over which the running mean was taken.

In practice, as Bilham explained, what was actually logged for each day was the minimum duration - the quickest clocking-up in the course of that day - of each in turn (as far as was applicable) of the amounts 5, 10, and 25 mm. (The scheme was later extended to the additional categories 15 and 20 mm) The final counting, under each heading, was of numbers of days of attainment of such intensities as to have brought the durations below certain specified spans of time.
It was shown that when this had been carried out there was a clear relationship between the time-spans (t hours) and the numbers of days (n) per decade :-

$n = 90\ t$ for rainfalls of 5 mm,
$n = 15\ t$ " 10 mm,
and $n = 0.9\ t$ " 25 mm,
approximately.

Only a few sizes of time-span, however, had been specified under each heading. The smallest under the first heading was 6 minutes ; the smallest under the second heading was 15 minutes ; and the smallest under the last heading was 60 minutes. Moreover the instruments in those days could not always yield records reliably resolvable within 6 minutes, and such readers as were not happy about the data under that particular heading had to turn to a 15-minute heading instead. Thus the foregoing formulae were from only a few fixes, not from an elaborate formal regression analysis. None the less they were combined in a single formula relating n and t to the rainfall r, and so long as it was not extrapolated unduly far outside some such ranges as

1 to 1000 in n,

0.1 hr to 2 hr in t,

and 0.2 inch (5 mm) to 1.0 inch (25 mm) in r,

this Bilham formula certainly proved its worth. Extrapolation to 24 hr in t received serious consideration, and Bilham also essayed big extrapolations in n, notably to $n = \frac{1}{4}$ and $n = \frac{1}{16}$; but apart from immediate use of the latter for defining the "noteworthy" and "remarkable" and "very rare" categories his extrapolations were of course speculative.

Some of the twelve stations, for one reason or another, left the list ; but some later starters came in. By 1960 the number in England and Wales that were featured in Table 6D and had been featured similarly throughout 35 years was just six, and meanwhile there were four runners-up. Summaries like Bilham's

had been compiled from time to time and had served simply to bear out what Bilham himself had brought out, and the latest long-term job of that kind covered 1926-1960 - a 35-year term comparable with the 1916-1950 term already used in other contexts. Certain headings (notably "5 mm") had been dropped from the published tables after 1936 : and although the "5 mm" heading reappeared twenty years later there were further changes among the duration sub-headings, with the result that only five of the original combinations of r and t survived, namely :-

(i) 10 mm (0.4 inch) within ¼ hr
(ii) " " ½ "
(iii) " " 1 "
(iv) 25 mm (1.0 inch) " 1 "
(v) " " 2 " .

The headings "25 mm within 5 hr" and "25 mm within 24 hrs" survived until 1955 ; and out of the set that had newly appeared in 1937 a few were still in use in 1960. Data under the heading "25 mm in 24 hr" receive special mention.

Under headings (i) to (iii) the numbers of station-days per station-year - or, as is handier for getting rid of the decimal point, per station-century - turn out to have been just about as close to Bilham's formula as his 1925-1934 data :

	r = 0.4 inch (1 cm)		
	t = ¼	½	1 hr
Days per century (10 n), from 1926-60 data	35	71	140
" " " 1925-34 "	38	77	140
" " " formula	37	73	146

Under headings (iv) and (v) the 1926-60 figures come out exactly the same as for 1925-34, respectively 10 and 15, while the formula gives 9 and 18. Under the heading of "25 mm within 24 hr" the 1926-55 figure was 234, as against 227 for 1925-34. The formula gives 214. Under such headings as were introduced in 1937 and have survived, the 1937-60 figures prove to have been well fitted by the formula except for a considerable shortfall displayed under the heading of 10 mm within 5 minutes. It will be recalled that the first twenty years of this 24-year term were summarised in Table 6F of British Rainfall 1957, and the feature in question was discernible at that stage if attention was focussed upon the nine English stations and Holyhead. The number of station-days per station-century for 10 mm within 5 minutes appeared to be of the order of 3, whereas the formula gave 12.

II. LATER DEVELOPMENTS, AND A FRESH APPRAISAL OF THE BILHAM TREATMENT'S SCOPE

By 1956 there had been introduced into some of the standard rain recorders of the Meteorological Office a type of strip-chart mechanism whereby the time-scale was magnified about twelvefold so as to allow heavy rain to be read quite accurately from minute to minute. With this "open scale" technique it was planned to extend Bilham's manner of treatment not merely over further long terms of years but also down to durations of a minute or two. (It was planned, too, to extend it to many more stations in order to spot the regional variations : this aspect, however, is for separate discussion) In view of this and of new developments in urban storm-water drainage engineering in this country the standard form of table of numbers of days of attainment of specified multiples of 5 mm of rain within specified durations underwent changes ; the "5 mm" heading was re-introduced ; and the layout which duly appeared as Table 6D in British Rainfall 1956 became the new standard.

At the time of writing this, the standard layout has been filled in for a nearly constant set of stations, right up to 1962 ; and in order to get as many years as possible the whole seven years 1956-62 have been analysed. It may be observed, from the separate annual Tables 6D, that although successive years' lists of stations have not been identical they have been very nearly so. In fact if we omit West Malling

(which was in the published list until 1959 only), and if we also leave out Greenwich for 1960 and 1961 (this station not having previously been in the list), then the rest of the list for England, as far as Acklington, becomes a list of 20 stations comparable with the 1925-34 group or with the 1926-60 group. Averaging over them we get a 14-station-decade summary in which the numbers of station-days under the various headings are reducible, on multiplication by $\frac{10}{14}$, to numbers per station-century. A really comprehensive 1961 compilation, together with a correspondingly comprehensive 1956-60 summary and a comprehensive 1962 compilation to follow, can of course yield more than 14 station-decades of data. But the 14 are analysed first. There has subsequently been a very comprehensive sifting-out under the 3-minute, 2-minute and 1-minute headings, covering the highest and rarest intensities of all, the upshot of which (as is to be described) has been greatly to strengthen the hint that the Bilham formula goes off the mark.

Under the headings of clocking up 5 mm within 15 minutes and within 10 minutes, first of all, the numbers of station-days per station-century come out at 229 and 143 respectively in close accord with Bilham's 223 and 148. Moreover under the heading of 10 mm within 15 minutes the number is 36, in good accord with Bilham's 37. But the numbers under the headings of 10 mm within $\frac{1}{2}$hr and within 1 hr are respectively 79 and 158 which are nearly 10 per cent higher than Bilham's ; and a corresponding scaling-up seems to be called for by the figures under nearly all of the higher headings :-

NUMBERS OF STATION-DAYS PER STATION-CENTURY

	by Bilham's formula	by Bilham's formula multiplied by 10/9 approximately	from 1956-1962 data
15mm within ½ hr	22	25	24
" " 1 "	44	50	49
" " 2 "	89	100	96
" " 4 "	178	200	233
20mm " 1 "	18	20	21
" " 2 "	36	40	44
" " 4 "	72	80	87
" " 8 "	144	160	203
25mm " 1 "	9	10	12
" " 2 "	18	20	21
" " 4 "	36	40	37
" " 8 "	72	80	80
" " 16 "	144	160	183

In the earlier notation, n being the number of days per decade, the suggested scaling-up would be thus

$n = t$ for rainfalls of 25 mm,

$n = 2t$ " " 20 mm,

$n = 5t$ " " 15 mm,

and if this were extended to smaller rainfalls we should have

$n = 16t$ for rainfalls of 10 mm,

$n = 100t$ " " 5 mm,

while from the original formula it is easy to verify that by extrapolating down to 0.04 inch (1 mm) and scaling up like this we get approximately

$n = 1500t$ for rainfalls of 1 mm.

In passing, it is worth noting that all of these, except the last, are quite well fitted by the formula

$$n = t\, r^{-20/7}$$

where r is the rainfall in inches. This is simpler than Bilham's formula

$$n = 1.25\, t\, (r + 0.1)^{-3.55}$$

though on extrapolation down far below r = 0.2 inch (5 mm) Bilham's formula continues to give fairly realistic orders of magnitude whereas with r = 0.04 inch the $tr^{-20/7}$ formula would indicate attainment of 1 mm within 1 minute on nearly 170 days per decade.

Within its limitations the $tr^{-20/7}$ formula serves as a rough guide to how the three variables may best be graphed. A three-dimensional representation on logarithmic scales is simply a sloping plane ; and a specified level of frequency n (or equivalently of return period, $\frac{1}{n}$ decades) cuts the plane along a certain straight line, a graph of log r against log t. Such a line must in general become a curve if the r or t scale is transformed, say from logarithmic to linear ; and in particular there will exist curves for n = 1, $\frac{1}{4}$ and $\frac{1}{16}$. Curves like this from the Bilham formula, rather than from the $tr^{-20/7}$ formula, were actually constructed by Bilham and have been in familiar use ever since 1935 for dividing heavy falls of rain into categories, though their physical truth is open to doubt. Similar curves for comparatively short return periods (i.e., for quite large n) have been used a lot ; and some attention has been paid to where - on the graph of r against t - they cease to be physically meaningful.

They are thought to be still fairly sound from t = 0.1 hr to 24 hr with n of the order of 20 to 30 ; but then with further increase of n (i.e., with more common-or-garden rainfalls, not specially heavy) their range of validity probably shrinks down from t = 24 hr until when n has risen to more than 1000 (i.e., on the average, twice-a-week stuff) they have become rather meaningless except perhaps for rainfalls of the order of a millimetre and durations of the order of an hour.

Bilham's curves were originally not drawn closer to t = 0 than t = $\frac{1}{12}$ hr. So their manner of approach to the origin did not need thinking about. Nowadays, however, we need to look into it carefully. With a formula like $n = tr^{-20/7}$ each n-curve of r against t can in principle come as close to the origin (r = t = 0) as we like. Physical common sense, at the same time, says that the shorter the time-step the more nearly constant is the rain's intensity during the course of it, the effective fluctuations being all longer in time-scale ; and it is intuitive that the slope to which the curve's direction tends, on approach to the origin, will be related to n in a unique way requiring that whatever relationship exists between r and t shall be such that rt^{-1} then depends on n only. With the formula $n = tr^{-20/7}$ this clearly cannot be so ; and still less can it be so with any n-curve from the Bilham formula.

What is worth noting about the Bilham formula, however, is that when r = 0.1/(3.55 - 1), namely when the rainfall

is 0.0392 inch (almost exactly 1 mm), any n-curve has a slope of exactly 1 when plotted on logarithmic scales of r and t : that is to say that the curve which forms the graph or r against t on linear scales then has as its tangent a line through the origin. Suppose we adopt this straight line, right down to the origin from the 1 mm level, as a substitute for the awkward bottom part of the downward-extrapolated Bilham curve. The result, although composite and therefore analytically not so tidy, has the double merit of being smooth (in the sense that its slope undergoes no jump) and of straightening out towards the origin as is ultimately necessary physically (as discussed above), even if it is not strictly realistic physically for the straightening to be invariably from the 1 mm level regardless of whether or not this is at a sufficiently small t.

The composite n-curve, constructed thus, will here be known as the straightened-off version of the Bilham curve.

III. TRIAL OF THE GUMBEL TYPE OF TREATMENT

It would seem that when r is given, n varies as t. To put it another way, the return period, 1/n, varies as the intensity, r/t. If this simple rule is to be extrapolated, how far can we take it ? Already there have been hints of its failure. What rule, if any, takes over ?

The requisite short-duration intensity data have hardly sufficed - at the time of writing - for a firm answer. Can physical principles help ?

As the intensities which are under discussion here are daily peak values, all the statistics are essentially statistics of extremes, more or less in Gumbel's sense.* Admittedly these intensities, having been not merely smoothed but taken with respect to r rather than to t, can hardly be expected to behave statistically quite like the unsmoothed or raw ones ; and even the latter are notoriously difficult either to sort out by observation or to infer satisfactorily from physical principles. So it is by no means obvious, a priori, what kind of statistics of extremes should be looked for. It might be better to smooth t-wise rather than r-wise, considering that we are dealing with peaks clocked up during specified regular intervals of time (viz., days). But the taking of t as variable, with r fixed, enables us to start off with the beautifully simple law that n varies as t ; and conventionally the statistics are laid out that way anyway. We therefore set out with the simple rule

---

* See, for example, Statistics of Extremes, E.J.Gumbel's comprehensive book on the subject, published in New York by Columbia University Press in 1958

of direct proportion of return period 1/n to intensity r/t. Whether plotted on linear scales or on log-log, this duly yields straight lines : and we certainly get sets of points conforming roughly to them in practice. In general, any breakdown of the rule will introduce curvature : and if this occurs just when extrapolation is about to begin we can hardly expect to get much further. Can we, by transforming any of the scales, straighten out this curvature, even though curvatures must then be introduced into what was formerly straight ?

One of Gumbel's treatments supplies an answer. This is the well-known "double exponential" rule which has often been applied to the distributions of return periods of the peaks attained by certain kinds of meteorological and hydrological variables in the course of successive intervals - days, months, years. There is a special graph paper for it ; and so long as the periods of return are reasonably long in comparison with the size of basic interval - a good many days, say, if we are dealing with daily peaks, or a good many years if we are dealing with yearly peaks - this graph paper is to all intents and purposes semi-logarithmic, the log scale being of the return periods themselves. The linear scale is of the peaks whose distribution is being assessed ; and if the distribution happens to be of the type under discussion the graph is a straight line. Gumbel has shown that this must be the case, under certain conditions, if the peaks are of something whose own frequency distribution is of an exponential type, and in so far as this is widely true (though the details need not be known) the Gumbel or semi-logarithmic type of graphing can be a powerful aid in extrapolation.

In our case, the peaks are the daily peaks of intensity, which are our r/t ; and the periods of return are 1/n in decades, 10/n years. Rain intensities may well have frequency distributions exponential in type, at least in the sense that the distributions tail off that way. Whether Gumbel's conditions are satisfied by our kind of data is not so clear. Hourly rates, for example, might not be strictly admissible : a single day cannot contain a sufficiently large sample of statistically independent hourly rainfalls to ensure that the sample's peak value fits Gumbel's requirements. Four-minute rates, on the other hand, may well obey his rules just as soundly in the context of daily peaks as is the case, say, with daily river-flow rates in the context of annual peaks - the number of four-minute intervals in a day being about the same as the number of days in a year. Even then, it is possible that the daily peaks themselves do not wholly enjoy statistical independence : stormy days are apt to come in bunches. But on the whole we might reasonably hope for a roughly straight line if we plot our r/t against the logarithm of 1/n, or (simply by reversing direction) against n itself as marked on semi-logarithmic graph paper. This is exactly what has been done with the 1956-62 data. The plotted points for such data as have already been parallelled by Bilham's formula now duly lie along curves, whereas those which refer to the highest and rarest intensities of all - notably under the headings of clocking up 5 mm within 5, 3, and 2 minutes - clearly indicate a straightening out.

Under the heading of 10 mm within 5 minutes the figures are such as to indicate a similar straightening. It was remarked earlier, in connection with data for 24 station-decades (1937-60), that under this particular heading the average number of station-days of attainment per station-century had been of the order of 3. The number from the 1956-62 data has been 2. Now the next of the durations under the 10 mm heading is 15 min., for which the number has been quoted already as 36, in good accord with Bilham's 37. The respective intensities, r/t, differ by $\frac{4}{3}$ mm/min., and the frequencies are in the ratio 36/3 (having logarithm 1.08) if we adopt the figure of 3, or 36/2 (having logarithm 1.25) if we take the 2 instead. It would seem, then, that the frequencies undergo roughly a tenfold change when the value of r/t alters by a little more than 1 mm/min.

Under the 5 mm heading, meanwhile, the 10 min. and 2 min. sub-headings represent r/t values differing by exactly 2 mm/min., and the numbers of station-days per station-century are respectively 143 and about 3, the ratio of which has a logarithm of about 1.7. This result parallels the other. Moreover the numbers under the 5 min. and 3 min. sub-headings are respectively 51 and 14, which are almost exactly in line with the 3 and 143 on the semi-logarithmic plotting. The frequencies evidently change about fivefold for every 50 mm/hr change in r/t. Equivalently, they change by a factor of 8/3 for every 30 mm/hr or $\frac{1}{2}$ mm/min.; and the breakaway from adherence to (or parallelling of) Bilham's formula

appears to occur at intensities of the order of 30-40 mm/hr or just over ½ mm/min. It is worth noting, too, that although, under the 15 mm heading, the number - 24 - under the 30 min. sub-heading has been shown to accord with the scaled-up version of the Bilham formula, the figure under the 15 min. sub-heading is 9, as against the scaled-up formula's 12. The ratio 24/9, being 8/3, is in fact exactly what corresponds to the ½ mm/min. difference in r/t on the logarithmic scheme ; and the breakaway starts at about 15mm/30 min., i.e., at about ½ mm/min., as before. There is even a hint of the same phenomenon under the 20 mm heading, too, though the data do not go beyond 20mm/30 min. The data under the 25mm heading do not reach 30mm/hr, so their failure to provide even a hint of a breakaway is not surprising. The shortest duration that has been logged under this heading is 1 hr, and we have already expressed doubt about fitting hourly rates into Gumbel's scheme anyway.

The upshot, thus far, seems to be that the simple rule of direct proportion of n to t (r being given), or of 1/n to r/t, breaks down at intensities of ½ mm/min. or so, to be superseded by a logarithmic relationship such that the return period lengthens fivefold for every 50 mm/hr further increase in r/t. This represents considerable deviation away from what is got by extrapolating the Bilham or $tr^{-20/7}$ type of formula, but it is quite in line with what might be expected of Gumbel's law. The deviation is particularly noteworthy under the heading of clocking up 5 mm within 2 min., inasmuch as Bilham would indicate 30 station-days of attainment per station-century whereas the logarithmic rule, fitting the available data from about 400 station-years, indicates only 3.

Under the heading of 5 mm within 1 minute, the 400 station-years yield no recorded instances whatever. Since not all of the stations had the open-scale facility it is admittedly conceivable that a duration of only 1 minute might have occurred and been put down as 2 min. (or possibly 3 min.) because of being practically indistinguishable on the chart. The shortest of all the minimum durations clearly recorded on an open-scale chart, to the writer's knowledge so far, has been 1.3 minutes. If the 'nil return' under the 1 min. heading is taken at face value we should infer a return period of at least 4 centuries (subject, of course, to the usual Poisson-distributed uncertainty of how to interpret a single count of a very rare event anyway). As r/t is 300mm/hr (being 5mm/min.), and as the return period at 150mm/hr has been shown to be of the order of one-third of a century, our logarithmic rule would suggest three successive fivefold lengthenings from about $\frac{1}{3}$, namely to something like 40 centuries. From our 'nil return' we have said "at least 4". There is no inconsistency.

The heading "1 mm within 1 minute" is new at the time of writing, and it can of course scarcely be used except where the records have open scale. Less than a station-century of material is available, so far. The average number of station-days of attaining 1 mm within 1 min., per station-century, looks like being of the order of 200 to 400. Bilham's formula points to about 225, and the scaled-up version indicates 250, this being the value of $10n = 15000t$ with $t = \frac{1}{60}$ hr.

Attainments of 1 mm within shorter durations have not been analysed at open-scale rain-recording stations in general, and so there is as yet no direct indication of the alignment of the intensity-frequency graph. But an indirect hint is available. An instantaneous or practically unsmoothed intensity of 100 mm/hr was attained on just 36 days in the 30 years 1933-1962 at Kew Observatory, whose intensity-frequency relationships under other headings are known to be not untypical of English stations in general. The latter would therefore seem hardly likely to clock up more than about one day per annum, in the long run, for attaining a whole millimetre at 100 mm/hr. This would constitute a lengthening of the return period by a factor of the order of 2-4 from the value when r/t (in mm/hr) was 60. Would this be consistent with Bilham ? The answer is no ; Bilham's formula would lengthen the return period merely in the ratio 100/60. Would it be consistent, on the other hand, with the logarithmic law, whereby the 40 mm/hr change in r/t ought to lengthen the return period by a factor of $5^{40/50}$? The answer is yes, the factor being nearly 4.

Earlier, when considering how to relate r to t when given the value of n, it was proposed to straighten off all the Bilham n-curves from the 1 mm level to the zero level. Wherever the Bilham type of formula has now been shown to give way to the logarithmic or Gumbel law, this proposal needs to be modified. It is simply proposed, instead, to apply a corresponding straightening-off to the appropriately amended n-curves. So far as we are concerned with relating n and r and t together at a typical or 'average' English station, the job is then finished.

IV. STUDIES OF SYSTEMATIC EFFECTS OF LOCALITY : LOCAL OR REGIONAL PRONENESS TO HEAVY RAIN

Two paragraphs from Bilham's original article in British Rainfall 1935 are worth quoting in full.

"In the course of the professional work of the Organisation," he wrote,"we receive many enquiries for information in regard to the most intense falls which have been recorded in specified areas, the information being required for purposes connected with the design of sewage or drainage discharge works. It is very natural for an engineer responsible for such works to take the view that the experience of the past is the best guide for the future. That view leads the engineer to call for information in regard to the area in which he is interested.
I now wish to suggest that it may not altogether be wise to base everything on purely local observations. One thing is very clear, namely that the incidence of heavy rains in short periods is very fortuitous. A particular area may for years enjoy an apparent immunity from intense rains as compared with some neighbouring district, but there is good reason to believe that in the majority of cases such immunity is due to pure chance rather than to real climatological characteristics.* For example in the 10 years 1925 to 1934, Kew Observatory experienced 4 falls of 1 inch in 60 minutes or less, but there were only 3 such falls in the whole of the 50 years 1878-1927.

---

* An apparent immunity may also arise merely because of the absence of recording raingauges in a particular area

Between 1878 and 1925 Kew Observatory enjoyed a remarkable immunity from this particular class of heavy rains, followed by an excessive frequency in the 10 years 1925 to 1934. I am personally convinced that the best estimate of the real frequency of rains of 1 inch in 60 minutes or less at Kew Observatory, or anywhere else in the drier parts of England, is that obtained by calculation from the equation ... namely about once in 11 years. The important point about this equation is that it is based on experience not only at Kew Observatory but at many other stations.

"'But,' it may be argued,'surely the liability to intense rainfalls cannot be the same for all localities.' The answer to that objection is that the existence of variability from place to place in respect to such rainfalls is very likely, but the information available up to the present is altogether insufficient to enable us to define the areas in which the liability is greater or less than the average. From the data ... the only thing that one can be certain about is that areas with a high annual average rainfall are more liable than areas with a low annual average rainfall, to <u>prolonged</u> heavy rains. The really intense rains lasting no more than an hour or two seem to be distributed without regard to the annual average rainfall. On the evidence available up to the present, he would be a bold man who would base designs involving a large capital expenditure on the assumption that his particular area was less subject to intense rainfalls than the hypothetical 'average station' to which our calculations apply".

Now that three further decades have passed since those days it may well be asked whether information has become sufficient by now to distinguish the above-average and below-average areas. By 1960, as has been stated, the requisite frequency data had been amassed for no less than 35 years at each of six stations in England and Wales (Wales being represented, however, by Holyhead only), together with a coverage of most of the same 35 years at each of four further stations in England. The nine English stations referred to were (1) Camden Square, (2) Croydon, (3) Kew, (4) Farnborough, (5) Cranwell, (6) Plymouth, (7) Boscombe Down, (8) Manchester, and (9) Southport. The first three were all in what is now Greater London ; the first five were those which covered the whole 35 years from 1926 ; and the first eight were still in the current list in 1961, though in certain instances there had had to be some slight local changes in identity of source of record. As well as these, there was a record from Lympne, in Kent, until 1953 ; and the Mildenhall record, which is still going strong, dates from 1935. Lastly but by no means least, there are the Scottish stations of Aberdeen and Renfrew and Leuchars, and the Lerwick and Eskdalemuir observatories ; and a record has been running from 1927 at Aldergrove in Northern Ireland. The requisite data from some of the foregoing stations had actually started in 1925 and had therefore qualified for Bilham's 1925-34 list ; but Lerwick and Manchester and Boscombe Down, like Aldergrove and Mildenhall, were later starters, and it has been decided to relate everything to the 35 years from 1926.

Reference has already been made to Table 6F of British Rainfall 1957 which gave a summary, station by station, for the 20 years 1937-56. All the foregoing stations, except Lympne and Southport (which ended their records before 1956), appeared in that. At first sight it sufficed for the purpose of showing up regional highs and lows in proneness to heavy rains ; and in fact this was attempted by firstly focussing attention on the six particular columns for which the 20-year numbers of days would be not less than 7 by the Bilham formula (thereby immunising against the sway of pure chance through the numbers in the rarer categories, which could still be wildly unrepresentative of true long-term values) and then expressing each figure as a percentage of Bilham's, followed by an averaging of the six percentages to yield an over-all or general percentage at each station in turn. The general percentage turned out to be between 70 and 130 at each of the English stations except Boscombe Down (where it was less than 60) and also at Eskdalemuir. At Leuchars it was only 51 ; and at the remaining stations - Holyhead, Aldergrove, Renfrew, Aberdeen and Lerwick - it was less than 40. Even among those figures that had been meaned over 20 years, however, and which had not been so small as to run too obvious a risk of being largely at the mercy of chance, there were some striking differences from their 1926-60 counterparts. Under the heading of 10mm within 60 min., for instance, for which - as we have seen - the annual number of days of attainment at a typical or 'average' station was 1.4 over 35 station-decades, the actual number clocked up at Aldergrove was 35 in the 34 years 1927-60 although only 12 in the 20 years 1937-56, the annual mean thus going up to 1.03 from 0.60.

Of the five headings which survived unchanged throughout the 35 years 1926-60, this particular one has already been shown to yield by far the largest counts. The figure of 1.4 corresponds to 49 in the 35 years, and although on the basis of Poisson probability the "98 per cent confidence limits" are then still as far apart as about 35 and 70 we can at least claim almost certainly to be able to gauge the true long-term answer's order of magnitude. The 35-year annual average figures for various stations under this particular heading have accordingly been compared :-

STATION	MEAN ANNUAL NUMBER OF DAYS OF ATTAINING 1 CM OF RAIN WITHIN 60 MINUTES	
Camden Square	1.31	(93 per cent. of 1.40)
Croydon	1.49	(106 " ")
Kew	1.63	(116 " ")
Farnborough	1.69	(121 " ")
Boscombe Down	.93	(66 " ")
Plymouth	1.87	(133 " ")
Cranwell	1.34	(96 " ")
Manchester	1.67	(119 " ")
Southport	.97	(69 " ")
Holyhead	1.09	(78 " ")
Eskdalemuir	2.00	(143 " ")
Renfrew	.60	(43 " ")
Leuchars	.94	(67 " ")
Aberdeen	.77	(55 " ")
Lerwick	.37	(26 " ")
Aldergrove	1.03	(74 " ")

This inter-comparison, like the earlier one, shows percentages ranging between about 70 and 130 at each of the English stations except Boscombe Down (where it remains peculiarly low), and again quite a high figure at Eskdalemuir, while the rest continue to run low - particularly Lerwick. Compatibility between the actual 35-year totals and the corresponding Bilham figure, on the assumption of approximately a Poisson distribution of probability, is obvious for the three Greater London stations which head the list (and which, indeed, average out amongst themselves to the Bilham figure almost exactly), and likewise for Farnborough and Cranwell and Manchester. Plymouth, however, is less readily compatible, and the remaining stations are hardly compatible with Bilham at all. At the time of writing this text there are actually data up to the end of 1963, and it is interesting to find that they shrink Plymouth's average whilst simultaneously raising Holyhead's, making them both more compatible with Bilham. Southport, of course, is no longer available ; and so we see Boscombe Down alone as the odd man out, so to speak, from the English group. Since nothing has been found wrong with the data the inference is that the uniform applicability of Bilham's formula (or any fairly close approximation thereto) is really too sweeping a generalisation even in the south of England. There can evidently exist "sub-Bilham" localities in what is at first sight a "Bilham" region. Conversely, the high figure for Eskdalemuir shows that a locality with values even higher than Bilham's can exist in what seems a "sub-Bilham" region. Conceivably the majority of upland localities are such. Under such headings as "25mm within 24 hr",

of course, an association of large frequencies with large average annual rainfalls in general (and with upland rainfalls in particular) has long been recognised, as Bilham pointed out ; and it can in fact be fairly well mapped out in so far as it reflects the already well-established pattern of frequencies of attaining an inch of rain in a "rainfall day". Further, as we have seen, the Bilham formula fits thirty station-decades of data fairly well, even under this heading. Strictly speaking, however, these hardly count legitimately as "heavy falls in short periods", notwithstanding having been published regularly in British Rainfall until 1955 in the chapter bearing this title.

In view of a natural association of the heaviest short-period rainfalls with severe thunderstorms, an article by A.F.Crossley and N.Lofthouse on the distribution of severe thunderstorms over Great Britain (Roy.Met.Soc.,Weather, XIX, p.172, 1964) is pertinent, though it is doubtful whether it really throws fresh light on what has been said about regional applicabilities of the Bilham type of formula. In particular its hint of high values in upland regions is but slight. It includes a reference, however, to the regional distribution of numbers of days of thunder (by no means the same thing as severe thunderstorms) ; and although it is recognised that this is not quite so appropriate a variable there have been efforts elsewhere - notably by V.K.Collinge in a paper on "The frequency of heavy rainfalls in the British Isles" (Part 2) published in Civil Engineering and Public Works Review, 56, p.497, 1961 - to exploit what is known about this variable.

Collinge also incidentally quoted a formula due to D.J.Maclean, originally published in Surveyor in 1945, which at first sight would seem to have foreshadowed the present development of a logarithmic relationship between intensity and frequency. In the very regime where the latter seems to be established, however, namely at intensities beyond about ½mm/min., Maclean's version needs to have its constants altered considerably.

Between 1956 and 1962, to help to fill the enormous gaps in the national network of open-scale rain recorders designed ultimately to reveal the regional highs and lows of frequency of high intensities, many Local Authorities installed such recorders and started logging minimum durations in the manner of the Meteorological Office stations. This enterprise has been co-ordinated by the Meteorological Office and has already yielded more than a station-century of the kind of statistics discussed here. Results up to now have resembled those that have been discussed but they are almost all post-1959, i.e., too recent to yield even a hint of the long-term answers at individual stations. Background to this enterprise has been outlined in Road Note No.35 of the Road Research Laboratory of the Department of Scientific and Industrial Research, published by H.M.Stationery Office in 1963 under the title of A guide for engineers to the design of storm sewer systems (prefaced by the Report of the Joint Committee on Rainfall and Run-off of the Road Research Board and the Ministry of Housing and Local Government).

V. STUDIES OF THE SMOOTHING EFFECT OF AVERAGING OVER VARIOUS SIZES OF AREA

Heavy falls of rain in short periods, in our usual sense of the term in this country, are well-nigh invariably from showers or storms that are so patchy or localised as to be sensitive to the process of averaging over quite small areas - a process of smoothing that may be described as spacewise. Moreover, because of their brevity, they are sensitive, as we have seen, to being averaged over quite short intervals of time - a process of smoothing that may be called timewise. From a single raingauge, with its relatively tiny area of exposure, the records are practically unsmoothed spacewise. If they are only read daily at a fixed time of day, then in effect they are smoothed timewise over 24-hour steps ; and those steps are not sliding but fixed. Autographic records, on the other hand, can be analysed unsmoothed, or nearly so.

Until fairly lately, no statistics of heavy falls in short periods had ever been able to have account taken, systematically, of smoothing spacewise. The only kinds of rainfall statistics that had been smoothed spacewise were those that had first been smoothed timewise very considerably - over whole days or months or years. A few special occasions may have been smoothed spacewise after only a few hours' smoothing timewise ; but certainly there had not been systematic <u>frequency</u> analysis, the spacewise smoothing being hard to assess unless gaugings are not only well synchronised for a very long time but also profusely distributed. Although this country's rain-gauging network is fairly profuse by world standards in general it has remained far too sparse

to yield firm statistics on frequencies of rains that are smoothed not only over short durations but over small areas too; and the Meteorological Office has for several years been helping to fill this gap in our knowledge by running synchronised networks of open-scale recorders in each of two localities - a flat locality around Cardington in Bedfordshire and a hilly one near Winchcombe in Gloucestershire. The scheme has been to map out the intensity patterns not merely from hour to hour but actually from minute to minute over various sizes of area down to less than half a square mile. At Winchcombe, at the time of writing, the fieldwork is still in progress, whereas the Cardington data have undergone part of their final analysis. Intensities of rains which were localised in the form of heavy showers, analysed in steps of two minutes, have undergone smoothing spacewise ; and in the long run they appear to be thereby toned down, so to speak, from their raw or unsmoothed values - which we may describe as 100 per cent -to something near 80 per cent when the area is a square kilometre, and to something near 60 per cent when the area is 10 Km^2 or four square miles. It will of course be appreciated that rain which is not of a showery or localised nature will not be so sensitive to such smoothing, and even the shortest and sharpest showers will respond less to smoothing spacewise if they have been smoothed timewise first over longer steps than two minutes. The toning-down then will be less marked.

The background to this enterprise, like that of the Local Authorities' open-scale rain recording scheme, has been outlined in the Road Research Laboratory publication referred to. Further results will become available at the Laboratory from the Meteorological Office and will probably be published after they have become complete.

RAIN INTENSITY FREQUENCY GRAPHS

From a 7 year (1956-62) assessment averaged over stations in England ;
From a 35-year (1926-60) assessment averaged over a group of 10 stations; } as shown in the data table
From Bilham's formula, multiplied by 10/9 ; } by appropriately marked boxes
From Bilham's original formula } enclosing the relevant figures

NUMBER OF STATION-DAYS OF ATTAINMENT PER STATION-CENTURY

RAIN INTENSITY
(averaged over a finite step of rainfall amount or of time-duration, as specified on the appropriate curves)

Return period 1yr
Return period 10yr
Return period 100yr

1mm within 1 minute
5mm within 15 minutes
10mm within 10 minutes
15mm within 4 hr
20mm within 8hr
25mm within 16hr
25mm within 24hr
60 minutes
30 minutes
15 minutes
10 minutes
5 minutes
3 minutes
2 minutes
2hr
4hr
8hr
1hr
BILHAM 10/9
BILHAM

50
100 mm/hr

LOG 3 CYCLES X 10ins. & 1 INCH
GRAPH SHEET No. 5233

TABLE OF NUMBERS OF STATION-DAYS OF ATTAINMENT OF SPECIFIED AMOUNTS OF RAIN WITHIN SPECIFIED SIZES OF TIME-STEP

from published and unpublished statistics compiled by the Meteorological Office from various stations' autographic rain records (including some with "open-scale" facilities) and averaged over stations in England as specified in the table's left-hand margin

Totals for the first 20 stations in Table 6D of British Rainfall

		1 mm within	5 mm within					10 mm within				15 mm within					20 mm within					25 mm within					
		1 min.	2	3	5	10	15 min.	5	15	30	60 min.																
		$\frac{1}{60}$ hr	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$ hr	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{2}$	1 hr	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4 hr	$\frac{1}{2}$	1	2	4	8 hr	1	2	4	8	16	24 hr
	mm/min.	1	$\frac{5}{2}$	$\frac{5}{3}$	1	$\frac{1}{2}$	$\frac{1}{3}$	2	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{24}$	$\frac{5}{12}$	$\frac{5}{24}$	$\frac{5}{48}$	$\frac{5}{96}$	$\frac{5}{192}$	$\frac{5}{288}$
	mm/hr	60	150	100	60	30	20	120	40	20	10	60	30	15	$7\frac{1}{2}$	$3\frac{3}{4}$	40	20	10	5	$2\frac{1}{2}$	25	$12\frac{1}{2}$	$6\frac{1}{4}$	$3\frac{1}{8}$	$1\frac{9}{16}$	$1\frac{1}{24}$
excluding West Malling	1956				10	33	53	1	6	22	45	4	5	11	28	57	4	7	10	32	60	4	6	13	27	44	
	1957				17	35	54	0	12	22	37	2	3	12	23	47	1	3	10	19	28	1	3	6	9	21	
	1958				13	36	54	0	14	21	42	1	7	13	26	53	2	5	13	18	46	3	7	10	20	47	
	1959				11	28	41	1	9	14	22	2	8	12	14	33	4	7	10	14	30	6	6	7	13	30	
excluding Greenwich	1960				12	42	59	0	7	20	41	2	7	14	23	74	0	3	12	18	62	1	4	9	22	57	
	1961				6	17	36	0	3	5	17	1	3	3	8	24	2	3	3	9	26	2	3	4	9	28	
excluding Camden Square	1962				2	10	23	0	0	7	17	0	1	4	13	39	0	1	4	11	32	0	0	3	12	29	
7-year totals, 1956-62					71	201	320	2	51	111	221	12	34	69	135	327	13	29	62	121	284	17	29	52	112	256	
Ditto, multiplied by $\frac{100}{7 \times 20}$ (i.e. reducing to 100 station-years)					51	143	229	2	36	79	158	9	24	49	96	233	9	21	44	87	203	12	21	37	80	183	
1960-63 totals from Local Authorities 'open-scale' stations (altogether 100 station-years)			3	...	34	100	161	1	16	46	101	2	6	22	61	142	1	8	18	52	....	2	7	17	....	....	
"Bilham"		223	30	45	74	148	223	12	37	73	146	11	22	44	89	178	9	18	36	72	144	9	18	36	72	144	216
Ditto, × 10/9, approx.		250	33	50	83	166	250	13	40	80	160	12	25	50	100	200	10	20	40	80	160	10	20	40	80	160	240
35-year (1926-60) 10-station (Holyhead + 9 stations in England) mean values reduced to 100 station years								3*	35	71	140											10	15				234
1957-62 totals over about 400 station-years			about 12	56																							
Ditto, reduced to 100 station years			about 3	14																							
1956-64 total over 57 station-years		170×																									
Ditto, reduced to 100 station-years		300																									

*based on data for 1937-60 only